Labor Supply over the Life Cycle

An Age-Period-Cohort analysis

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Summary

Labor force participation rates in Norway have increased steadily over the last few decades. Especially, female labor force participation has increased significantly. Comparing Norway to other European countries and the United States shows similar trends in the countries. The reasons for the increase are several, for instance technological development, education and the need for women to have their own income.

Differences in the labor supply over the life cycle are a natural effect of age, but the labor supply also varies over time and according to birth year. In the absence of period and cohort effects, life cycle patterns should be constant over time. In the absence of cohort effects, the labor supply should increase but the increase would be equal for different cohorts.

The thesis uses income data from the Norwegian tax administration covering over 40 years, from 1967 to 2011. It investigates the development of the labor force participation for men, women and the general working age population and focuses especially on separating effects from age, period of time and birth year. The age-period-cohort model aims to identify and interpret *age effects*, the effects of differences in ages of individuals over time; *period effects*, the effects of differences in time periods of the measurement of the observations; and *cohort effects,* the effects due to differences in year of birth and some shared life events. The descriptive evidence shows that the increase in female labor force participation occurred both at the same time for different age groups and at a different time within age groups. This suggests that also period and cohort effects play an important role in the labor force participation of the population.

Different methods to solve the age-period-cohort problem are discussed. The first approach, setting age, period, or cohort effects to zero, suggests that age effects are the most important and that period and cohort effects seem to complement each other. The Constrained generalized linear model (CGLIM) approach, where adjacent ages are set equal, depends highly on which age group chosen to be equal and is for that reason not an appropriate method to solve the problem. The Deaton-Paxson approach suggests that period effects increase labor supply while cohort effects decrease labor supply for the total sample. A downward-sloping cohort effect seems unreasonable. Because of the clear break in the time trend of labor supply of women, which violates a central assumption for the Deaton-Paxson methodology, this may not be an appropriate method to approach the problem. Comparing the period effect estimated by the Deaton-Paxson approach to unemployment rates, shows that the

estimates for men correlates inversely with unemployment. The estimates for women do not follow the participation rate in the Labor Force Survey or correlate with the unemployment rate. The maximum entropy method estimates positive period effects for both men and women. The cohort effect is increasing for females and decreasing for males. Comparing the period effect estimated by maximum entropy to the unemployment rate shows that both male and female participation rates correlates inversely. A final approach seeks to avoid the problem of collinearity by using a proxy-variable that correlates with either period or cohort effects. Unemployment can be used as a proxy for time because it reflects the labor market conditions over time. When using unemployment rates as a proxy for period, the cohort effect is very high, and it seems like some of the period effect is absorbed by the cohort effect. According to Oreopoulos et al. (2012) may labor market conditions in the early years of the carrier have large long term effects on earnings. Because many take higher education and go into their first real job around the age of 25, GDP deviation at age 25 reflects the labor market conditions when people enter the labor market. Since the labor market conditions at the time of entering the labor market is assumed to have a long term effect, GDP deviation at age 25 is used as a proxy for cohort effects. When using this as a proxy for cohort, the period effects are very high. This confirms that the period and cohort effects are absorbed into the other effect when one is left out.

All methods show that the age effect is an important component of the changes in labor supply over the life cycle. They estimate approximately the same life cycle pattern, showing that the labor supply increase from the 20s and the 30s when it is normal take education and get children, reach a top in the 50s and then the labor supply decline until reaching retirement age. The different methods also find distinct period and cohort effects but they do not agree on how large these effects are nor in which direction they are pointing. Considering the limitations of the different approaches the maximum entropy approach seems to be the most reliable approach, suggesting that female period effects increase the entire period, except for a flat period around the banking crisis in the beginning of the 1990s. The effect from the banking crisis is much more visible for men, with a large drop in labor force participation. Female cohort effects increase for cohorts born between 1920 and 1950, reflecting the entrance of the housewives into the labor market. The male cohort effects are decreasing the whole period. The plots of period effects and unemployment rates shows that for men especially, the estimated period effect on labor force participation and unemployment rates are inversely related. The female plot shows a similar story but the effects are less distinct.

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Any errors and inaccuracies in this thesis are my own responsibility.

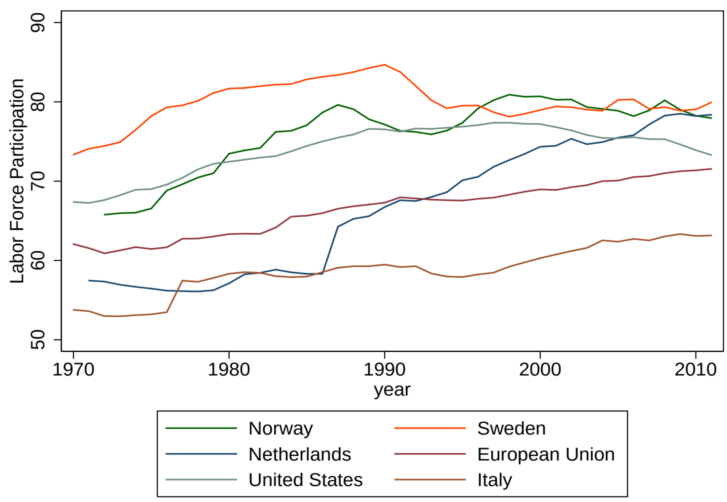
Contents

1 Introduction

Labor force participation rates in Norway have increased steadily during recent decades.[1] In 1970, the participation rate was about 61,4 percent of the population aged 15 to 74 years. Today it is almost ten percentage points higher, at 71 percent.[2] A part of this increase is because women entered the labor force during this period. Another part of it may be due to increased longevity and better health among the elderly. Older men especially are working longer than before.

Figure 1 shows the labor force participation rates for people aged 15 to 64 in selected European countries in addition to the United States and the European Union.[3] The figure shows that the labor force participation in Norway is high compared to other European countries. The participation rate has decreased somewhat since the financial crisis.

Figure 1. Labor force participation in Europe and the United States



Notes: The figure shows the labor force participation rate for women aged 15 to 64 years from 1970 to 2011 in selected countries/areas: Norway, Sweden, Netherlands, the European Union, the United States, and Italy.
Source: OECD

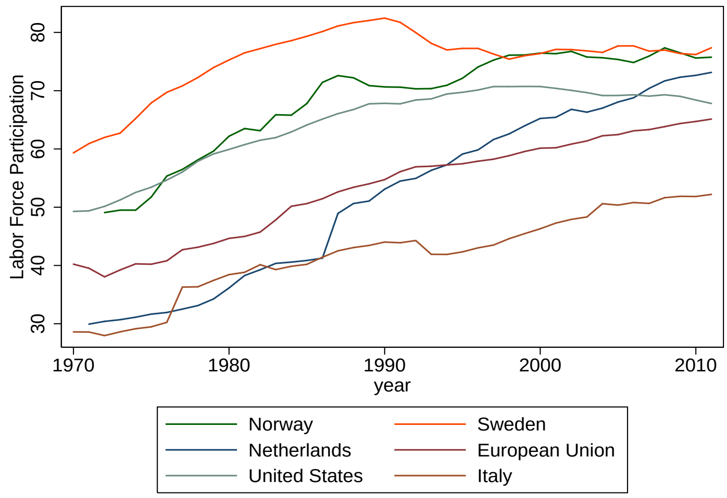
[1]Labor force participation and labor supply are used interchangeably in the text. A person participates in the labor force if he or she is either employed or unemployed. The labor force participation rate gives the fraction of the population that is in the labor force and is defined by: Labor force participation rate = Labor force / Population (Borjas (2013), page 22).

[2]Statistics Norway (2014a)

[3]For the cross-country comparison, I use people aged 15-64 years due to data limitations in some of the comparison countries. When I look at Norwegian data separately, I will use the age span 15-74 years.

Figure 2 shows the labor force participation rate for women aged 15 to 64 years in selected European countries and the United States. The labor force participation rate has increased in all countries, but the differences are still large. Sweden had the highest participation rate in 1970, while Netherlands and Italy both had very low participation rates. In 2011, Sweden and Norway had the highest participation rate. Netherlands has seen an large increase in female participation, while in Italy female labor force participation is still low compared to the other countries included here.

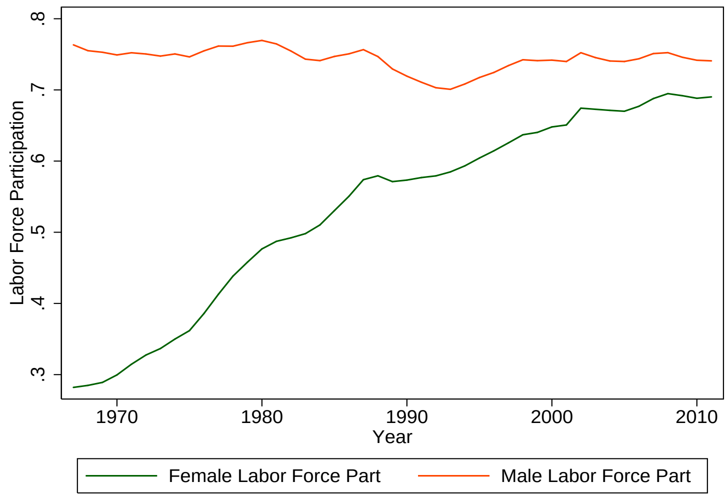
Figure 2. Labor force participation, women



Notes: The figure shows the labor force participation rate for women aged from 15 to 64 year in some selected countries/areas: Norway, Sweden, Netherlands, European Union, United States and Italy from 1970 to 2011.
Source: OECD

Figure 3 shows the labor force participation rates for men and women in Norway from 1967 to 2011 using register data. It displays some distinct features of the labor market and the Norwegian society in general over the last 40 years. In the 1950s and 1960s, described as the "golden age of the housewives," only a small number of women participated in the labor force. And while most women at that time were home, taking care of the house and the children, women today participate in the labor market at almost the same rate as men. While the participation rates for males have been more or less constant over this time period, the rather remarkable rise in female participation has taken place during only one generation.

Figure 3. Labor force participation rates in Norway



Notes: The figure shows the labor force participation for men and women aged 15 to 74 years, from 1967 to 2011. Labor force participation rate is estimated using income data from the Norwegian tax administration. See Section 4 for details.

It seems to be a common opinion that the women who entered the labor force came from nowhere and suddenly started working (Koren (2012), page 11). Data from the Norwegian Time Use Survey clearly shows that this view is debatable.[4] The Norwegian Time Use Survey shows how much time people use at work, at household work, on education, on personal care, and leisure activities. The results from 1970 compared to 2010 show that the labor supply of women has increased over the last 40 years. At the same time, the labor supply of men has been almost constant. At the same time the time used on household activities has decreased by about two hours for women and increased almost one hour for men. The time women spent on household activities in 1970 was even more than the time men spent on work. The time spent on education has not changed much during the period for men, but the time women spent on education has increased. Time spent on personal care has decreased somewhat and time spent on leisure activities increased somewhat for both men and women. Some of the activities women performed as housework in 1970 are publicly offered services today, and these occupations have a significant over-representation of women (Koren (2012), page 12).

[4]The purpose of the Time Use Survey is to collect information about how people spend their time on different activities. The survey is conducted every tenth year and is used especially to show development in gender equality, Statistics Norway (2012).

Table 1. The Time Use Survey

	1970	1980	1990	2000	2010
A. Women					
Work[a]	1.56	2.23	2.48	2.59	3.01
Household activities[b]	5.55	4.46	4.22	3.56	3.5
Education[c]	0.17	0.31	0.33	0.27	0.28
Personal care[d]	10.36	10.19	10.1	10.11	10.24
Leisure activities[e]	4.59	5.56	6.01	6.2	6.11
B. Men					
Work[a]	5.29	4.4	4.3	4.34	4.1
Household activities[b]	2.13	2.26	2.36	2.41	3
Education[c]	0.23	0.3	0.29	0.22	0.27
Personal care[d]	10.19	10.11	9.58	9.46	9.59
Leisure activities[e]	5.21	6.08	6.21	6.28	6.18

Notes: This table shows the main activities from the time use survey from 1970, 1980, 1990, 2000 and 2010 for women and men.
[a]: average hours spent on work per day and time spent traveling to/from work. Average is taken over 365 days a year.
[b]: activities including house work, maintenance, family care, shopping and services, travel related to household activities and other household activities.
[c]: time spent on education
[d]: time spent on night sleep, meals, and other personal care
[e]: time spent on sport and outdoor activities, culture and entertainment, television viewing, travel, reading, and socializing.

Source: The Time Use Survey, Statistics Norway (2012)

A number of explanations for why women entered the labor force have been suggested. The feminists highlight on the right and need for women to have their own income. Technological development decreased the time needed for cleaning and cooking. For instance, the washing machine became common in the 1950s. In 1972 about 72 percent of Norwegian households had a washing machine.[5] Access to contraceptives also decreased the number of children. Increasing education made the alternative cost of staying home higher. International trade made goods cheaper: in particular, clothes and the need for making one's own clothes decreased. The thesis does not aim to give causal evidence of any of these mechanisms. The thesis does not aim to give causal evidence of any of these explanations.

The empirical question this thesis will try to answer is; how much of the increase in labor supply over the last 40 years is due to age, period, and cohort effects? The

[5]The Norwegian Folk Museum: http://www.norskfolkemuseum.no/Utstillinger/Faste-utstillinger/Teknologi-i-hjemmet/Hygiene/

age, period, and cohort effects on labor force participation are separated using an age-period-cohort (APC) model. An APC analysis is a model that aims to identify and interpret *Age effects,* the effects of differences in ages of individuals over time; *Period effects,* the effects of differences in time periods of the measurement of the observations; and *Cohort effects,* the effects due to differences in year of birth and some shared life events.

Differences in the labor supply over the life cycle are a natural effect of age, as people supply labor when they finish school, get children and retire when they get old. But the labor supply also differs according to time and birth year. In the absence of period and cohort effects, the age pattern should be the same over time. In the absence of cohort effects, the labor supply should increase and be equal for different cohorts. The data show a major increase in female labor supply over the last 40 years, and the increase was both at the same time for different age groups and at different times for equal age groups. This implies that period and cohort effects also play an important role for labor force participation in a society.

Two problems arise when one tries to separate age, period, and cohort effects. The first problem is that each of the dummy sets sums to one. This problem is easy to solve by setting the first age, period, and cohort dummy equal to zero and interpreting the others in comparison to this. The second problem is that the explanatory variable is perfectly collinear. By definition, $Year = Cohort + Age$. Hence, there is a perfectly linear relationship between age, period, and cohort. This problem can be overcome either by imposing restrictions to avoid the problem or by measuring age, period, or cohort indirectly by using another variable that correlates with one of the variables.

The relevance of this approach was first recognized in early epidemiological studies where descriptive evidence suggested that early life conditions rather than current conditions influenced mortality rates. One of the first well-known studies was done by Frost (1940), and suggested that early life conditions were more important than current life conditions on tuberculosis mortality rate. Frost plotted tuberculosis risk against age and birth year and showed that those born in the earliest decades had the highest risk of tuberculosis. Kermack et al. (1934) investigated death rates in Great Britain and Sweden and found that there were irregularities in death rates due to earlier life conditions. The usefulness of the observations demonstrated in these early studies showed their applicability in other studies where health, mortality, or longevity is an important component.

The thesis will use the APC model to separate age period and cohort effects to explain the increase in the labor supply over the last 40 years. The rest of this paper is organized as follows: Section 2 will provide an overview of the main literature on labor supply and retirement, and the APC model and the different approaches to solving the problem. Section 3 discusses a model of labor supply to compare with the empirical results. Section 4 provides an overview of the data used and some descriptive evidence. Section 5 explains the APC problem in depth and discusses different methods used to solve the APC problem. Section 6 shows the empirical results of the different methods explained in section[6] 5. Section 7 concludes.

2 An overview of the literature

Over the past decade, a number of papers have contributed to the empirical literature on labor supply over the life cycle and retirement decisions. The theoretical framework used has typically been some version of the standard life-cycle model of labor supply, as in Cahuc, Carcillo, and Zylberberg (2014), which will be discussed in detail in Section 3. Recently, more and more researchers have become interested in the APC approach, which was first introduced by Ryder (1965). Ryder, as a sociologist, was especially concerned about the cohort effects and how changes in birth cohorts pose a threat to society but also provide the opportunity for social transformation. He was concerned with developing measurement techniques designed to provide data on cohort effects. The APC approach has traditionally been used mostly by sociologists and demographers, but is also relevant to many research topics within the field of economics.

2.1 Labor supply and retirement

One part of the literature focuses on empirical research on labor supply. There is a substantial amount of both Norwegian and international literature on both labor supply over the life cycle and retirement and pension reforms.

Heckman (1974) used the life-cycle model and wage growth to explain the difference in the consumption of leisure and goods by age. He discusses the results in comparison to the empirical results of Thurow (1969) and Nagatani (1972). Altonji and Paxson (1985) used micro data to estimate the intertemporal substitution of labor supply. Goldin (1989) looked at the life cycle labor force participation of women.

The decision of labor supply over the life cycle is dependent on different incidents

[6] Estimations are conducted in Stata 13.0, using OLS, and the ado file -apc- for linear apc-models generated by O'Dea (2012) to calculate Deaton-Paxson estimates and maximum entropy estimates.

in life. When people get older, the society's pension system is of great importance. Different pension systems give different incentives for labor supply and retirement. Ellingsen and Røed (2006) looked at the effect of reduced payroll tax on retirement age. Haider and Loughran (2001) found that it is the most educated, healthiest, and wealthiest among the elderly who tend to work in old age. Those who choose to work do so for a comparatively low wage. Bråthen and Bakken (2012) found similar results using Norwegian data. In addition to good health, they found that economic incentives and the time of one's spouse's retirement are important factors in one's decision to retire.

The implementation of a new pension system in Norway started in 2011. The reform included new rules for annual income, age adjustment, and flexible outlet from age 62 to 75 years, making it more flexible than the old one. It is also possible to draw a pension and continue working without a cut in pension. These changes do have had a great impact on labor supply among the elderly. The traditional function of pensions was to provide income when people stopped working; now the system includes elements of choice between work and leisure. These changes in the pension system have both direct financial effects and indirect effects via labor supply behavior.[7] Brinch et al. (2015) study how the Norwegian pension reform of 2011 affected labor supply and pension-claiming behavior of older workers. They find that the pension system affects workers affiliated with an early retirement scheme and others in a different way. They found large positive effects on employment rates of contractual pension scheme (AFP) workers, while labor supply responses among non-AFP workers occurred only along the intensive margin, and employment rate remain unchanged.

The labor market conditions when entering the labor force may also be important. Oreopoulos et al. (2012) estimated the long-term effects of entering the labor market in a recession. He found that the cost of recession for new graduates was substantial and unequal. The higher the pre-existing inequality in earnings in the labor market is, the larger the persistent rise in inequality due to initial shocks predicted by the model. Those in the lowest quartile of predicted annual earnings are most affected by higher initial unemployment conditions and experience permanent earnings losses. After one year in the labor market, earnings are about 15 percent lower with a five percentage point increase in the initial unemployment rate. After ten years, they remain 7,5 percent lower. By contrast, in the upper quartile, earnings are about 7,5 percent higher in the first year, but the gap falls to less than 2,0 percent after only four years. The median group experiences similar patterns. The changes in

[7] Regjeringen (2013)

unemployment and income can potentially affect labor supply.

The main change in labor supply in Norway the last 40 years has been due to increased female participation. Koren (2012) explains female attendance in the Norwegian economy and why the labor supply changed from 1950 to 2010. She goes through the economic consequences for households and society and explains both why and how women changed their workplace and how this increased productivity and gross national income. She also explains how taxes and social benefits changed as a result of the change in labor supply. Giuliano (2014) explains differences in labor market variables between men and women by cultural traits and beliefs about gender inequality in the society. Thus she introduces a different way to look at differences in the labor force participation between genders than what has traditionally been done in literature.

2.2 Age, period cohort analysis

This thesis relies on literature focusing on the theory of the APC model. The APC problem occurs because of the perfectly linear relationship between the variables: $Year = Cohort + Age$. Over the past few decades, several papers have used the APC model to address different research questions. The main part has been done by sociologists, but demographers, epidemiologists, and economists also have used this method to address different questions. At least as far back as Ryder (1965), it has been argued that the three effects can be separated. Ryder defined cohort as an aggregate of individuals within the same population definition who experience the same event within the same time interval. Different age groups are at different stages relating to education, work, and retirement, and different cohorts are exposed to different events relating to war, recession, and the relative size of the cohort compared to neighboring cohorts.

Mason et al. (1973) introduced an APC multiple classification model and suggested the constrained generalized linear model (CGLIM) as a means of estimating independent age, period, and cohort effects. Deaton and Paxson (1994) introduced a possible solution to the problem when they used the APC model to investigate savings growth and aging in Taiwan. They solved the problem by imposing that once the variable has been detrended, time effects sum to zero. This is known as the Deaton-Paxson normalization. Kupper et al. (1983) first introduced the principal component estimator. He also noted different sources for bias.

Browning et al. (2012) presented a new approach to the APC problem based on a maximum entropy method. They illustrated the method by using two different examples, US female mortality and UK female labor force participation, and also compared it to other methods. They found that the maximum entropy method described in Glenn (2005) reproduces patterns in age, period, and cohorts that are similar to both results based on different normalization methods and results from the intrinsic estimator. They concluded that maximum entropy methods provide a coherent and useful approach to the APC-problem.

Yang et al. (2008) proposed a new solution to the APC-problem, the intrinsic estimator (IE), which they argued is better than the CGLIM approach. This approach does not use regularizing identifying assumptions in the regular way but imposes some requirements on the geometric orientation of the parameter vector in the parameter space. This approach is further described in Yang and Land (2013a).

Luo (2013) claimed that he could show that IE cannot be used to recover true age, period, and cohort effects because IE, like CGLIM, imposes a constraint on parameter estimation that is difficult to verify using empirical evidence or theory. He argued that IE implicitly assumes a constraint on the linear APC problem. This constraint not only depends on the number of age, period, and cohort categories, but also has nontrivial implications for estimation. Because this assumption is difficult to verify in empirical research, IE cannot be used to estimate age, period and cohort effects. He argues that IE is no different than the constraint assumed in CGLIM except that age, period and cohort groups change. He also argued that IE is just an extension of the principal component estimator (PCE) introduced by Kupper et.al. (1983). IE adds nothing new to the question of how to solve the dependency problem. PCE/IE is just one possible solution of the infinite number of solutions for the undetermined problem. Hence, PCE/IE should not be regarded as the true solution any more than other constrained estimators. Yang and Land (2013b) answered the paper by Luo and claimed that he had misunderstood the intrinsic estimator and that his paper was nothing more than an "algebraic demonstration of a situation – identical linear or nonlinear algebraic trends in the effect coefficients for all three temporal dimensions."

Another method for solving the APC problem is the Bayesian cohort model. Fukuda (2006) used this approach, first introduced by Nakamura (1986), which built on the basic idea of empirical Bayesian modeling for linear problems explained in Akaike (1980).

Heckman and Robb (1985) discussed the APC problem in earnings equations. The paper considers the general issue of whether longitudinal data can be used to solve the earnings equation or its error in variables analog. One approach to the problem is to assume that specifically measured variables proxy the underlying unobserved variables. Another approach is to use longitudinal data in an effort to break the exact linear dependence between age and experience. According to Heckman and Robb, an even better way to formulate the problem is to write the earnings equation as a latent variable model. They claim the real problem is not the linear relationship between age, period, and cohort, but how to find more and better proxies. Glenn (2005) summarized the field of cohort analysis well. He argued that cohort analysis has been used in little more than exercises and discussions of how we can use cohort analysis in a more productive way.

Jappelli (1999) estimated the age-wealth profile for italian households under two different identification assumptions about age, period, and cohort effects. The life-cycle hypothesis predicts that wealth increases up to retirement and declines smoothly thereafter. It also predicts that growth takes place across generations but not over the life of a single individual. The model makes strong assumptions about the shape of the age-wealth profile as well as the size of the cohort effect.

Biørn et al. (2013) used the APC model to investigate the increase in sickness absence in Norway from 1993 to 2005. They used a model with cohort-invariant time and age coefficients that identified the difference between any two differences with distance 1, and then interpreted the results in comparison to a reference age. They find that the age-adjusted absence propensity has risen significantly within individuals and can rule out the hypothesis that the rise in long-term absenteeism is a result of the entry of new cohorts into the labor market with weaker work-norms than the older. They can also rule out the idea that the rise results from integration of marginal workers with poor health.

The literature on the APC problem contains many different solutions, and the methods have been used and discussed in different settings. Different methods will be discussed in depth in Section 5.

3 A model of labor supply

To evaluate and interpret estimates of labor supply well, it is useful to have an underlying model to provide a context for comparison. An economic model will never paint a true picture of the real world and it will always rely on, sometimes questionable, assumptions. However, it gives us a useful simplification of a complex situation, and in this section an example is presented as a convenient framework for structuring the later empirical investigations.

The basic labor supply model, the neoclassical model of labor-leisure choice, is based on the assumption that the consumer decides how much labor he or she will supply depending on the choice between consuming goods or leisure in a single period.[8] In the real world, the decision of labor supply depends on more than the simple trade-off between consumption and leisure. In the simple model, people can choose how much they will work. In reality, rather few have the opportunity to choose exactly how much they want to work. In some sectors, like the service sector, part-time jobs are available, but in most sectors, there are mostly full-time jobs (37,5 hours a week). Another drawback of the model is that money is not the only source of utility achieved by working. Furthermore, work does not always give negative utility; some people like their job. In the model, more consumption always increases utility, which is not necessarily true. The time aspect is also a drawback of the simplest static model, but it can easily be expanded to a life-cycle model, where we integrate the choices of agents over the course of time.

3.1 A basic model of labor supply over the life cycle

There is no doubt that age has a large impact on labor supply. In addition, both time and cohort effects are important. Over time, social norms of what a normal workday are and when it is normal to retire change. This can be seen as a time effect. Also cohort effects play an important role when people are influenced by what age they are when various events happens. Therefore, it is natural to think that age, period, and cohort effects will have a large impact on the labor supply over the life cycle. Hence, an APC analysis may provide motivation to look at an intertemporal model of labor supply. The intertemporal model of labor supply includes age as an important factor.

Cahuc et al. (2014) describes a dynamic model starting in period 0 and ending in period T, with t=1,2,...T periods. In the model it is assumed full information (certainty) and that utility is temporally separable. Then the agents have preferences regarding consumption and leisure today and in the future, and they maximize utility by optimizing consumption and leisure in each period given an expected lifetime budget constraint. By working more today, they can trade some leisure today in return for additional consumption tomorrow. In addition, a great deal of evidence suggests that the age-earnings profile has a predictable path. Workers earn less at the beginning of their career; earnings peak around the age of 50 and then start to decline slightly. Thus, the price of leisure is relatively low for the young and old and is higher for persons in their prime-age working years.

Substitution and income effects differ depending on permanent and temporary wage changes and anticipated and unanticipated wage changes. In this model, the agents have the opportunity to save.

[8]Borjas (2013), page 27

The utility of the agent over the life cycle is given as follows

$$U = \sum_{t=1}^{t=T} U(C_t, l_t, t,) \tag{1}$$

where C is consumption, l is leisure, and t represents the utility obtained by the consumer in the course of period t. This restriction is included to obtain analytically simple and interpretable results. The index t is also used as an indicator for age, professional experience, or seniority of an individual. This representation of preferences is very restrictive and does not allow us to take into account, for instance, habits of consumption.

In this model, agents have the opportunity to save, and wealth in period t is given by

$$A_t = (1 + r_t)A_{t-1} + B_t + w_t(1 - l_t) - C_t \tag{2}$$

where all t $\geq$1.

A_t describes wealth in period t as a function of wealth in the last period, A_{t-1}, the interest rate, r_t, non-labor income, B_t, labor income, w_t, leisure, l_t, and consumption,C_t, in period t.

The consumer maximizes the intertemporal utility subject to a credit constraint, which is given by the sum of wealth in all periods. The Lagrangian maximization problem is

$$L = \sum_{t=1}^{t=T} U(C_t, l_t, t) - \sum_{t=1}^{t=T} \lambda_t (A_t - (1 + r_t)A_{t-1} - B_t - w_t(1 - l_t) + C_t) \tag{3}$$

where λ_t denote the Lagrange multiplier.

First order conditions with respect to C_t, l_t and A_t are

$$\frac{\partial L}{\partial C_t} : \; U_c'(C_t, l_t, t) = \lambda_t \tag{4}$$

$$\frac{\partial L}{\partial l_t} : \; U_l'(C_t, l_t, t) = \lambda_t w_t \tag{5}$$

$$\frac{\partial L}{\partial A_t} : \; \lambda_t = (1 + r_{t+1})\lambda_{t+1} \tag{6}$$

where λ represents the marginal utility of wealth.

Combining equation 4 and 5 gives: $\frac{U_l}{U_c} = w_t$, showing that given the assumption of separable utility, the marginal rate of substitution and the current wage are maintained every period. Equation 6 is the Euler equation and shows the relationship between the multiplier and the interest rate.

Solving this, optimal consumption and optimal leisure can then be written as

$$C_t = C(w_t, \lambda_t, t) \tag{7}$$

and

$$l_t = l(w_t, \lambda_t, t) \tag{8}$$

The labor supply is then $h(w_t, \lambda_t, t) = 1 - l(w_t, \lambda_t, t)$. This shows that the labor supply in period t depends on the current wage and the marginal utility of wealth in period t, λ_t.

Taking the logarithms of the Euler equation (equation 6) gives the law of motion of λ_t, which can be divided into a fixed individual effect and an age effect, common to all agents:

$$ln\lambda_t = -\Sigma_{\tau=1}^{\tau=t} ln(1 + r_\tau) + ln\lambda_0 \tag{9}$$

where λ_0 depends on all wages received during lifetime and $-\Sigma_{\tau=1}^{\tau=t} ln(1 + r_\tau)$ is the age effect.

This shows that λ_0 depends on all wages over the life cycle, and if T is sufficiently large, a shock to w_0 will affect the multiplier very little. By contrast, a permanent shock to all wages will have a larger effect on λ_0.

In the model, a permanent shock has no effect on the labor supply since the income effect and the substitution effect cancel each other out. However, a transitory shock will affect the labor supply through the optimal response of agents. If leisure is a normal good, an increase in the wage will increase the alternative cost of leisure and hence, increase labor supply. The income and substitution effect pull in different directions. The substitution effect decreases the time spent on leisure and increases the labor supply. Opposite, the income effect decreases labor supply because an wage increase, increase the total amount of income, and the agent can afford more leisure.

One limitation of this model is that it does not include the decision of training.[9] Training increases human capital and the age-earning profile of the individuals and hence labor supply decisions. At the same time there is a trade-off between leisure, working time and time spent on leisure, which is not included in this model.

[9] The cost of training and education includes expenses on study, potential loss of earnings, and psychologically costs arising from stress and difficulties with the study (Cahuc et al. (2014), page 198)

3.2 The decision of going into retirement

The decision of going into retirement can also be included in the model. When the agent decides whether to retire, he or she will maximize the utility of consumption and leisure, and take into account that retirement will reduce consumption.

$$Max_{C_{et},C_{rt},L_t} = [\Sigma_{t=\tau}^{s-1} U(C_{et}, l_t, t) + \Sigma_{t=s}^{T} U(C_{rt}, l_t, t) \tag{10}$$

subject to

$$A_t = \begin{cases} (1 + r_r)A_{t-1} + B_t(0) + w_t(1 - l_t) - C_{et} & , \tau \leq t \leq s - 1 \\ (1 + r_r)A_{t-1} + B_t(s) - C_{rt} & , s \leq t \leq T \end{cases} \tag{11}$$

where $B_t(0)$ is non-earned income when the agent is still working, $B_t(s)$ is non-earned income expected in period s≤t (including pension payments), C_{et} is consumption before retirement and C_{rt} is consumption after retirement. Equation 10 shows the wealth in period t given the choice of retiring.

In this model, the decision of retiring depends on initial wealth, the size of the pension, consumption, and wage. The consumer maximizes the total utility of consumption and leisure before and after retirement. In the real world, the decision of retirement typically also depends on the family situation, health, mandatory retirement age, and the nature of the work. Several studies have shown the importance of the health situation of the agent and the retirement decision of the spouse (see for example Bråthen and Bakken, 2012). The fact that such things are not included in the model is a huge drawback.

In 2011, a new pension reform was implemented in Norway. The reform made the system much more flexible than the old one. There were many reasons for the implementation of the new pension scheme. The main goals of the new system are (1) *to be economic and social sustainable, (2) to give a good distributive and equality profile, and (3) to build on easy and comprehensible principles*. The first, and perhaps most important, reason relates to the fact that we have a growing elderly population, which makes the costs of the old pension system too high. The new system offers a greater opportunity for a flexible pension starting at age 62, which is supposed to increase the incentives for working longer by increasing the yearly pension the longer one stays in the labor force. It is important to increase the possibility set of choices without decreasing the sustainability of the pension system. The pension system aims to reflect that the pension level also should reflect the expected number of years in retirement. In addition, the single individual should be confronted with the real socioeconomic cost of the choice of time of retirement. This implies that people with good health can stay longer in the labor force, contributing to society and withdraw a higher pension premium in later years when they decide to retire. People with illness or bad health with a lower expected lifetime can take out a graded or even full pension at a lower age. This will increase both the sustainability of the pension system and the utility of individuals. If people freely chose whether they want to retire or not, the pension reform expand the choice set, the pension indicates revealed preferences and increase total expected utility over the life cycle. On the other hand, the possibility of drawing a graded pension from a lower age may contribute to pushing people out of the labor force. If the employer use the opportunity of graded pension as a reason or justify that they fire older workers, then older workers may feel a pressure to retire earlier than they actually want.[10]

In the context of the model, the new pension system implies that one can take full or graded pension in combination with staying in the labor force. This is included in the third option in Equation 12.

$$A_t = \begin{cases} (1+r_r)A_{t-1} + B_t(0) + w_t(1-l_t) - C_{et} & , \tau \leq t \leq s-1 \\ (1+r_r)A_{t-1} + B_t(s) - C_{rt} & , s \leq t \leq T \\ (1+r_r)A_{t-1} + B_t(s) + w_t(1-l_t) - C_{bt} & , \tau \leq t \leq s-1 \end{cases} \tag{12}$$

where C_{bt} is consumption after retirement when the agent combine retirement with work.

3.3 How does the model fit reality?

In this model, increased non-earned income will reduce labor supply. An increase in wages will increase consumption possibilities (income effect) and at the same time increase the cost of leisure (substitution effect). The effect of a wage increase will depend on the size of the two effects. If leisure is a normal good, the income effect will dominate the substitution effect and a wage increase will reduce labor supply. In the case of women the last 40 years, the substitution effect seems to have dominated. The wage increase has increased the cost of staying home, inducing women into the labor force. This reasoning is consistent with the explanation that women entered the labor force owing to a higher alternative cost of staying home because of higher education.

[10]Regjeringen(2013)

4 Data and descriptive evidence

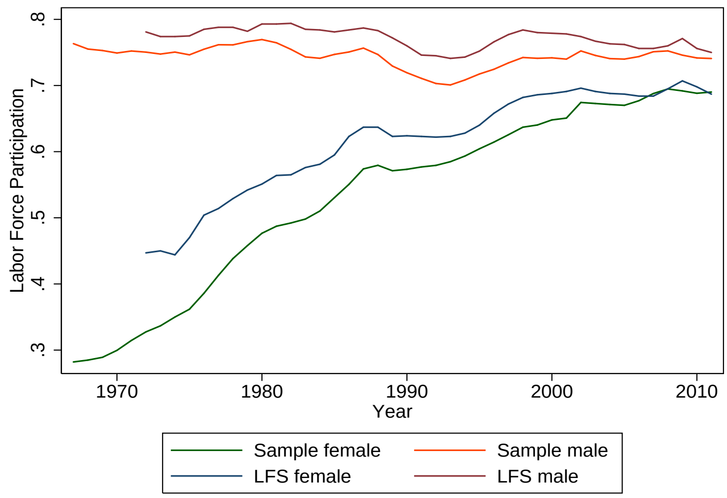
The empirical study of labor supply has large data requirements. Ideally, researchers need a complete overview of each individuals labor supply over the lifetime for people born in different cohorts. The data used in this thesis are income data from the Norwegian tax administration linked with administrative records of individual demographic characteristics and information on earnings to obtain a panel data set from 1967 to 2011. This data set is convenient to use because it is measured consistently for all years, and it is also the data set that spans the longest time period of the available registers. This gives a complete income data (pensionable income) for the population, with small measurement and reporting errors, such as attrition that is often occurring in survey data, compared to e.g. survey data that most studies use. Out of this data set, a random selection of 10 percent of the total sample is used.[11] This means that the data set cover income data for 10 percent the population born in 1900 or later for a period of 44 years. This sample can be used to calculate labor supply (as a function of income) and observe changes according to age, period, and birth year.

A challenge regarding the data is to obtain a consistent measure of the labor force participation over time. Since labor supply is not measured in the data, it must be approximated for by income. This definition can potentially be of importance for the results. In the Labor Force Survey, Statistics Norway, every person who works for at least one hour a week, are counted as a participant of the labor force. The World Bank defines labor force participation as the proportion of the population aged 15 and older that is economically active.[12] In this thesis, participation in the labor force is defined as having pensionable income at least equal to the threshold of the Norwegian Social Insurance scheme (G). Pensionable income includes the benefit gained by work (wages, fees), calculated personal income from industry, benefits that take the place of employment income (sickness), and certain social security benefits (rehabilitation, rehabilitation allowance, and temporary disability benefits).[13] The Norwegian Social Insurance scheme (G) is used to calculate national insurance and pension benefits. The amount is decided every year in the parliament; first time was in 1967, when the national insurance system was founded. The parliament has adopted guidelines for the regulation of the basic amount. The goal is for the amount

[11] A random selection of 10 percent of the total sample is used due to computational costs.
[12] Statistics Norway (2015), The World Bank
[13] Store Norske Leksikon

to have the same development as wages. The basic amount was 5 400 NOK per year in 1967 and 79 216 NOK in 2011. The labor force participation rate calculated by this definition and the labor force participation rate in the Norwegian Labor Force Survey (LFS) are shown in Figure 4. Comparing the labor force participation rate calculated from the data to the labor force participation rate in the LFS, shows that our participation rate is somewhat lower.[14] One reason is that those who worked only a few hours a week are not included. The difference between the two measures is larger in the 1970s and converges as we go forward in time. Since the difference is larger for women than for men, this suggests that the number of women who work only a few hours a week is declining.

Figure 4. Labor force participation rate, measured from register data and LFS



Notes: The labor force participation rate calculated from the sample and the labor force participation rate in the Norwegian Labor Force Survey (LFS).

4.1 Descriptive age-period-cohort analysis: Labor force participation by cohort

Table 2 provides summary statistics for the sample in 1972 and 2011. 1972 is the first year when unemployment was sampled and 2011 is the last year in the data set.

[14]The Labor Force Survey provides data on employment and unemployment and is collected by Statistics Norway. A person in the labor force is either employed or unemployed. An employed person is a person aged 15-74 who performed work for pay or profit for at least one hour in the reference week, or who was temporarily absent from work because of illness, holidays, etc. An unemployed person is a person who was not employed in the reference week but who had been seeking work during the preceding four weeks and was available for work in the reference week or within the next two weeks (Statistics Norway (2014b)).

During the period, the average age increased from 41 to 44 years. Average years of schooling increased from 12,1 to 13,4. The labor force participation rate increased from 53,7 percent to 74,2 percent. The unemployment rates was low in 1972, only 1,7 percent. This is most likely because women had the opportunity to work in the home. Today, the same women can report them selves as unemployed, which can explain a part of the large difference.

Table 2. Descriptive Statistics

	1972		2011	
	Mean	Std dev	Mean	Std dev
Male	0.50	(0.50)	0.51	(0.50)
Age	40.91	(16.75)	44.17	(15.44)
Birth year	1931.06	(16.75)	1966.84	(15.44)
Income	18916	(21370)	323670	(315441)
Participation rate	0.53	(0.50)	0.74	(0.44)
Education (year)	12.08	(2.41)	13.35	(2.68)
Unemployment rate	1.70	-	3.30	-
GDP deviation from trend at age 25	0.00	(0.09)	0.02	(0.08)
Observations	273 923		346 540	

Notes: The table provides summary statistics for the sample in 1972 and 2011.
Income is pensionable income, including the benefit gained by work (wages, fees), calculated personal income from industry, benefits that take the place of employment income (sickness) and certain social security benefits (rehabilitation, rehabilitation allowance, and temporary disability benefits).
Participation is defined as those who had pensionable income at least equal to basic amount (G), which was 7 900 NOK in 1972 and 79 216 NOK in 2011.
The unemployment rate is from the Labor Force Survey.
GDP deviation from trend is estimated by taking the log of the GDP and then removing the trend.

Figure 5, 6 and 7 show a plot of the labor force participation rate profile for cohorts spaced by five-years intervals, aged 15 to 74 years, for all men and women, respectively. The first cohort, born in 1930, was aged 37 in the first sample year. The youngest cohort, born in 1975 was 36 years old in the last sample year. The data follows some of the cohorts for 44 years, so the plot of the raw data provides a good cohort-specific picture of labor force participation over the life cycle.

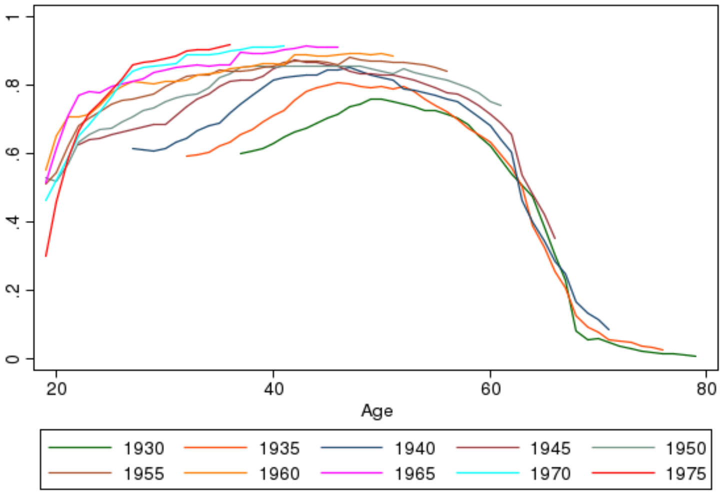
Considering the first cohort born in 1930, almost 60 percent of them participated in the labor force in 1967, looking at men and women together. Until they were 50 years old, their labor supply increased. Then it gradually decreased until retirement. In 2011, when this cohort reached age 71, almost none of them were working. Separating men and women in the 1930 cohort shows that the labor force participation rate of men was more than 80 percent in 1967, and then it started to decline after the age of 50. The labor force participation rate of women born in 1930 was just above 20 percent in 1967; it increased significantly until they were 50 years old, and then it declined. Thus, the increase in labor supply from this birth cohort is due only to the increase in female labor supply. Clearly, this pattern is consistent with a marked age effect (a hump-shaped participation rate over the lifetime), a time effect (an increase in participation over time), and a cohort-specific effect (an increase or decrease from one cohort to the next). If only one cohort were observed, these effects would be hard to disentangle, and it would not be possible to make any claim as to whether the increase in the participation rate was cohort specific, a pure age effect, or reflective of a common time trend that affected all cohorts in the period 1967–2011.

The cohort born in 1945, whose members can be followed for almost their whole work career. They are 22 years old in the first sample year and 66 years old in the last sample year, only one year from retirement age. In 1967, about 60 percent were participating in the labor force. Then they increased their labor supply when they were in their 20s and 30s and had a high labor supply during their 40s and 50s before they started to retire in their 60s. Also in this cohort, separating men and women shows that the labor supply of men was almost flat during the whole period, but the labor supply of women increased until they were 50 years old before starting to decline. This pattern shows the same as the 1930 cohort. The age effect follows the same pattern. The labor supply increase at the same time suggests that there is a period effect, and the labor supply of this cohort lies above the 1930 cohort the whole period, which suggests a cohort effect.

The last cohort, born in 1975, was 18 years old in 1993, and we follow them for 18 years. This cohort showed lower labor force participation than other cohorts until they were almost 30 years old. The LFPR increased a lot in this period, and from age 30, they had a higher labor supply than the rest of the cohorts. This change in trend can be seen as an education effect. From the 1970s it became more common to obtain higher education and hence enter the labor force later.

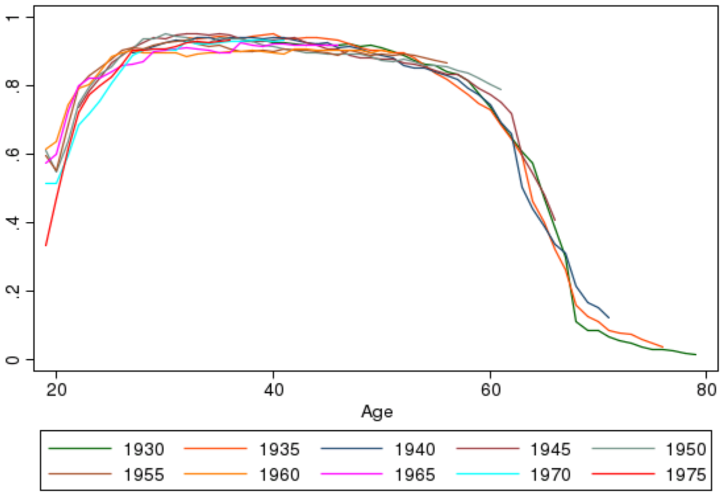
Considering all the cohorts, Figure 5 shows that there is an upward trend for all the cohorts during the period. This increase can be identified as the period effect. The jump between the cohorts, given age, can be interpreted as the cohort effect. This graphical presentation suggests that both period effects and cohort effects, as well as the age effect, play an important role in the increase in the labor supply the last 40 years.

Figure 5. Labor force participation against age, selected cohorts



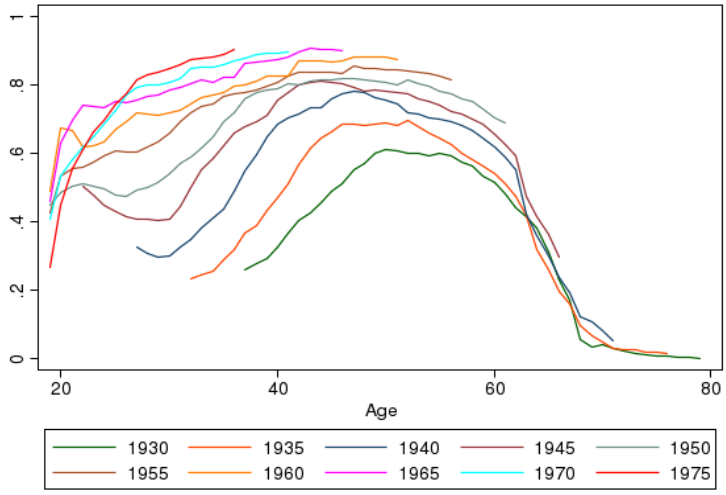
Notes: The graph shows the labor supply for different cohorts over the life cycle, estimated by using income data from the Norwegian tax administration.

Figure 6. Labor Force Participation against age, selected cohorts, men



Notes: The graph shows the labor supply for different cohorts over the life cycle for men separately, estimated by using income data from the Norwegian tax administration.

Figure 7. Labor Force Participation against age, selected cohorts, women



Notes: The graph shows the labor supply for different cohorts over the life cycle for women separately, estimated by using income data from the Norwegian tax administration.

Using descriptive evidence is a useful way to motivate further analysis. Even though these figures show the effects, they do not describe variations in the rates attributable to either age, period, or cohort. This is why statistical regression models are needed to understand the underlying effects and how they operate simultaneously.

5 Methodology

The descriptive evidence suggests that age, period, and cohort effects are likely to play an important role in the development of the labor force participation over time. However, the descriptive evidence does not deal with the key issue: how to separate these three effects. This section will discusses why it is an issue to estimate age, period, and cohort effects separately and explains different methods to deal with this problem.

5.1 Model specification: separation of age, period, and cohort effects

The APC-model aims to separate age, period and cohort effects;

Age effects refers to variations associated with chronological age groups. The variations arise from biological and psychological changes over the life cycle. This can be seen in the variations of the different regularities, such as schooling, employment, fertility, marriages, retirement, and so on.

Period effects are variations over calendar years, that affect all age groups at the same time. Examples are historical changes, environmental changes, economic expansions and contractions, pandemics and epidemics, public health interventions, and technological breakouts.

Cohort effects refers to changes across groups of individuals who experience an initial event, such as birth or graduation, at the same time. Birth cohorts are most commonly used in APC analysis. A birth cohort moves through life and experiences the same period effects at the same age. Thus, different birth cohorts, who experience different historical and social time events at a different stage in life, have diverse exposure to socioeconomic, behavioral, and environmental risk factors (Yang and Land (2013a)).

The model has two identification problems. The first problem is that each of the dummy sets sums to one. The second problem is the perfectly linear relationship between age, period, and cohort: $Year = Cohort + Age$. The first one can easily be dealt with by setting, for example, the youngest age group or the most recent cohort group to zero and then interpreting the results in comparison to this normalization (Browning et al. (2012), page 5). The second problem is a more demanding one.

The regression for labor force participation, by individual i, aged a, belonging to cohort c in period, p can be written as:

$$Y_{iapc} = \mu + \alpha_a A_{ap} + \beta_p P_p + \gamma_c C_c + \varepsilon_{iapc} \tag{13}$$

where $a = 1, ..., n$, $p = 1, ..., n$, $c = 1, ..., (p - a)$ and $c = p - a$. A, B, and C denote dummies for age, period, and cohort. A_a is set to one if person i is aged a at the end of year p. C_c is a dummy set equal to one if person i was born in year c and P_p is a dummy set equal to one if labor force participation is recorded in year p. Then Y is a binary variable which is equal to one if individual i born in year c at age a in year p, zero otherwise.

In matrix form,

$$Y = Xb \tag{14}$$

where Y is the estimated labor force participation and X is the design matrix consisting of dummy variable column vectors for the vector

$$b = (\mu, \alpha_1, ..., \alpha_{a-1,}\beta_1, ..., \beta_{p-1}, \gamma_1, ..., \gamma_{c-1})^T \tag{15}$$

The ordinary least square[15] estimator of the matrix regression 14 is: $\hat{b} = (X^T X)^{-1} X^T Y$

[15]Ordinary least squares is a method for estimating linear regression models where the goal is to

This is a linear probability model that estimates the probability of being in the labor force. The identification problem occurs because the design matrix[16] X is singular with one less than full rank, meaning that one column can be written as a linear combination of the others. This implies that the inverse ($(X^T X)^{-1}$) does not exist, and hence, we cannot obtain estimates of the coefficients. This results in a situation where there are infinitely many solutions to the matrix equation 14. Therefore, it is not possible to solve the APC model without imposing at least one extra constraint in addition to the reparametrization. This is the model identification problem of the APC analysis (Yang and Land (2013a), page 63)).

It is also possible to solve the APC problem without further restrictions under one of the following conditions: (1) Age and period are represented by dummies for a single year while cohort is represented by dummies for a decade. If the problem is represented sufficiently with this approach, then the problem can be solved by OLS (or similar). (2) People born in different months are distinguished, so they have a different age even though they are born in the same year. Then the problem is no longer collinear. For instance, if one person was born in January 1990 and one was born in December 1990, they will be 21 and 20 years old in January 2001. The identification is then variation in age between individuals born in the same year.

Both of these methods are inappropriate in this analysis. There is much that happens over a decade that refers to the birth cohort as those born in the same decade gives no meaning, canceling out the first suggestion. Information on birth month was unavailable at the time, canceling out the second suggestion.

5.2 Solutions to the APC problem

There are different ways to deal with this linear relationship. Glenn (2005) stated that a good method to solve the APC problem must

1. give approximately correct estimates more often than not, and
2. assess the credibility of the estimates by using theory and side information.

Several methods have been suggested to cope with the APC problem; some were already discussed in section 2. The easiest method is the reduced two-factor model. Another often used method is the constrained generalized linear model (CGLIM). Also, Deaton-Paxson normalization and the maximum entropy method are good

minimize the observed responses, and the responses predict the linear approximation of the data. It uses the observations in the data and tries to find the line of best fit.

[16]The design matrix is a matrix containing data on the independent variables to attempt to explain observed data on the dependent variable.

suggestions. In addition, one may avoid the problem by using a proxy variable for one of the three variables. This section will explain these methods in depth.

In addition, Yang and Land (2013a) introduced a new approach called the intrinsic estimator. This method gives approximately the same results as the other methods and it will not be explained further.

5.2.1 Setting age, period, or cohort effect equal to zero The easiest way to deal with the linear relationship between age, period, and cohort is to remove the collinearity by setting either age, period or cohort effects equal to zero. This is done for each variable in three different equations. Equation 16 shows an example where cohort effects are removed.

$$Y_{ap} = \mu + \alpha_a A_a + \beta_p P_p + \varepsilon_{ap} \tag{16}$$

Now it is not a linear relationship between the variables anymore, and the coefficients can be calculated using ordinary least squares (OLS).

The intuition behind the method is simply that if, by removing one variable, there is no change in the remaining two variables, then the removed variable is of little importance. By contrast, if the result of removing one variable is that the remaining two effects change a lot, it is natural to believe that the variable removed is important for the total effect. This method is intuitively simple, but it seems somewhat unrealistic that one of the effects has no impact at all.

5.2.2 Constrained generalized linear models (CGLIMs) Constrained generalized linear models were first introduced by Mason et al. (1973). These models are methods that take the form of placing at least one additional identifying constraint on the parameter vector. This can be done by constraining two age, two period or two cohort effects to be equal. For instance, the coefficients for age 29 and age 30 can be set equal. Then, $\alpha_{29} = \alpha_{30}$, the linear relationship is removed and OLS can be used to get coefficient estimates.

With this additional constraint, the model becomes just-identified, the matrix $(X^T X)$ becomes non-singular, and the OLS estimates has one single solution. Then OLS can be used to calculate estimates for age, period, and cohort effects.

Normalizing a pair of adjacent age effects is a commonly used approach to the APC analysis. The intuition behind the approach is that two adjacent age groups will in reality be as good as equal, so there will be no loss of information by setting them equal. The model then becomes just-identified and can be solved by regular estimation approaches.

The problem with this approach is how to decide which pair of adjacent ages that should be set equal. The effect on the parameters depends highly on this choice, which makes the estimation rely on prior or external information. In addition, all CGLIMs will produce the same level of goodness of fit to the data, making it impossible to use this as a criterion for choosing the best constrained model.

5.2.3 Deaton-Paxson normalization The Deaton-Paxson normalization approach was first introduced in Hanoch and Honig (1985), and was further described in Deaton and Paxson (1994).

The basic idea of the Deaton-Paxson normalization is to impose one extra parametric restriction so the APC model becomes just identified. First the variables are detrended, and then the restriction that time effect dummies are orthogonal to a trend and sum to zero is imposed.[17] The coefficients for age, period, and cohort can then be estimated using OLS.

The main problem with this approach is that many time periods are needed to be able to separate trends from transitory shocks. Since the data used here cover 43 years (the first observation is left out because of the first problem explained in section 5.1), there is no problem to separate trend and cycle.

Another problem is the assumptions about the trend. It is assumed that the trend is without any clear breaks. If the observations are characterized by having a break in the trend, the assumptions about the trend will be violated, and hence the results will be misleading.

5.2.4 Maximum entropy estimation The maximum entropy method was first used to solve the APC model by Browning et al. (2012). This is an information-based approach where the maximum entropy is used as a principle to address the problem. The approach is based on the belief that there is not enough information in the data to provide one unique solution. Instead of finding one unique solution, the maximum entropy principle provides a framework that can formalize the uncertainty in the model and estimate the most likely solution (Browning et al. (2012), page 10).

According to Conrad (2013), “The maximum entropy principle tells us to seek

[17]Detrend: The practice of removing the trend from time series (Wooldridge (2009)).

the probability density function such that certain constraints and use the density satisfying the constraints with the largest entropy".

The problem is $Y_{iapc} = \alpha_a A_{ap} + \beta_p P_p + \gamma_c C_c + \varepsilon_{iapc}$, where the sum of the APC-coefficients correspond to conditional expectations for all observable and non-observable combinations of age, period, and cohorts. The main idea is to parameterize the coefficients of the APC model, in terms of a probability distribution over the set of possible solutions. The probability distribution over the set of possible outcomes that has the maximum entropy allowed by the data is the most uninformative distribution possible. The entropy is the average amount of information contained in an event or a sample received from a distribution or a data stream. Hence, the entropy characterizes the uncertainty about a source of information and can be seen as a measure of uncertainty (Conrad (2013)). Choosing a distribution with lower entropy will assume information that is not in the data, and choosing a distribution with higher entropy violates constraints provided by the information that is processed in the data. The maximum entropy distribution best represents the current state of knowledge given the constraints – the conditional means. (Browning et al. (2012)).

To solve this, the model can be reparameterized as done in Browning et al. (2012).

The original problem is to solve equation 14 for the vector b. Instead, the problem can be transformed to choose the optimal probability distribution.

The first step is to represent the identified set by a hypercube

$$\Pi = \{b | b = Sp\} \tag{17}$$

where $S = [s_1, s_2, ..., s_j]$ and s_j are vectors representing the vertices of the cube and p is the vector of non-negative weights that sum to one and are used to form all of the convex combinations of the vertices (Browning et al. (2012), page 12). Using this means that b can be represented by Π, and we can insert equation 17 into equation 14. This gives

$$b = XSp \tag{18}$$

The problem is now to solve equation 18 for p instead of solving equation 14 for b. The problem is still undetermined, but can now be reinterpreted in a way that allows us to use the maximum entropy. The p_j´s have all the characteristics of probabilities, and the p vector can be treated as a probability distribution over the J multivariate outcomes of the matrix S. The reparametrization allows us to choose one probability distribution over another instead of one parameter vector over another. Then LaPlace's principle of insufficient reason can be used to claim that one probability distribution is better than another.[18] The idea is to choose a distribution that does not underlying favors one outcome over another subject to the requirements that the probabilities are non-negative, sum to one, and satisfy any data-based restrictions one might have.

The next step is to use the entropy measure: $H(p) \equiv \sum_j p_j log p_j$, introduced by Shannon (1948). It can be seen as a measure of uncertainty, where higher entropy corresponds to less information and more uncertainty. This measure is maximized when the probabilities are uniform. The constrained optimization problem is

$$max \sum_j p_j log_e p_j \; subject \; to \; b = ASp \tag{19}$$

which is a nonlinear optimization problem with one unique solution.

The solution to the maximization problem p* can be interpreted as a vector of probabilities over support points, and $b* = ASp*$ can be interpreted as the expected value of a discrete multidimensional random variable consistent with the entropy-maximizing choice of the underlying probability distribution.

A limitation of the maximum entropy model is that it is best applied in situations where the set of possible solutions is bounded, which is not always the case with the APC model. In our case, the dependent variable, labor force participation rate, is bounded, and we can use the maximum entropy method to solve the APC problem Browning et al. (2012).

Golan et al. (1996) also discussed the maximum entropy methods and argued that when there are multiple solutions, the maximally noncommittal choice is to select the probability with the minimum information, or equivalently, the maximum entropy.

5.2.5 A proxy variable approach Heckman and Robb (1985) argued that the real problem with the APC model is not the linear dependency but rather finding more and better proxies, better explanatory variables, and sharper behavioral models that eliminate the statistical problem. The idea is that the effect of the explanatory variable is captured through the proxy variable. For instance, to proxy cohort effects a variable that correlates with cohort, but not with age and time is needed. Suggested proxy variables for cohort can be unemployment in early years, number of vacancies when the cohort is entering the labor market, or deviation from GDP trend

[18] LaPlace's principle of insufficient reason: "If we want to assign probabilities to an event, and see no reason for one outcome to occur more often than any other, then the events are assigned equal probabilities" (Conrad (2013), page 1)

in early years of their career. The challenge with these proxies is that they don't necessarily capture desired variation in the cohort variable. Cohort effects consist of many different effects put together. Hence, a good proxy variable capture as much as possible of this variation. The literature provides some examples. For instance, macro economic conditions early in life (such as graduating during a recession) may have long lasting effects on labor market outcomes later in life, as in Oreopoulous et al. (2012) and Liu et. al (2012), the latter on Norwegian data. Giuliano and Spilimbergo (2014) find that large macroeconomic shocks experienced during the "impressionable years" between the ages 18-25 shape economic preferences later in life, similarly Malmendier and Nagel (2012) show that macroeconomic conditions early in life affect stock market participation later. Fagereng et al. (2013) use stock market returns during early ages as a proxy for cohort effects in a model for stock market participation.

It is also possible to use a proxy variable for period effects. Candidates here include unemployment over time or number of vacancies over time. The challenge is the same as with a proxy for cohort.

The main critique against this approach is that not all the variation associated with age, period, and cohort dimensions are accounted for by the proxy. One may ask whether it is possible to find proxies that are good enough.

In this thesis deviation from GDP trends at age 25 will be used as a proxy for cohort and unemployment at time t will be used as a proxy for period effects. GDP deviation from trend is estimated by taking the log of GDP per capita and then removing the trend.

When using a proxy for cohort effects, cohort can be replaced by GDP deviation in equation 4.1, and then regress:

$$Y_{iac} = \mu + \alpha_a A_a + \beta_p P_p + \gamma_c GDP_c + \varepsilon_{iact} \tag{20}$$

where GDP is deviation from GDP trend at age 25.

When using a proxy for period effects, period can be replaced by unemployment at time t and regress:

$$Y_{iapc} = \mu + \alpha_a A_a + \beta_p unemployment_t + \gamma_c C_c + \varepsilon_{iapc} \tag{21}$$

where $unemployment_t$ is unemployment rate at time t. The effects are explained in Section 6.5.

6 Results

This section provides solutions to the different approaches explained in section 5. The estimations are calculated using the same data set and definitions described in section 4.

6.1 Setting age, period, or cohort effects equal to zero

Figure 8 shows the results produced by normalizing respectively age, period, or cohort effects to zero.

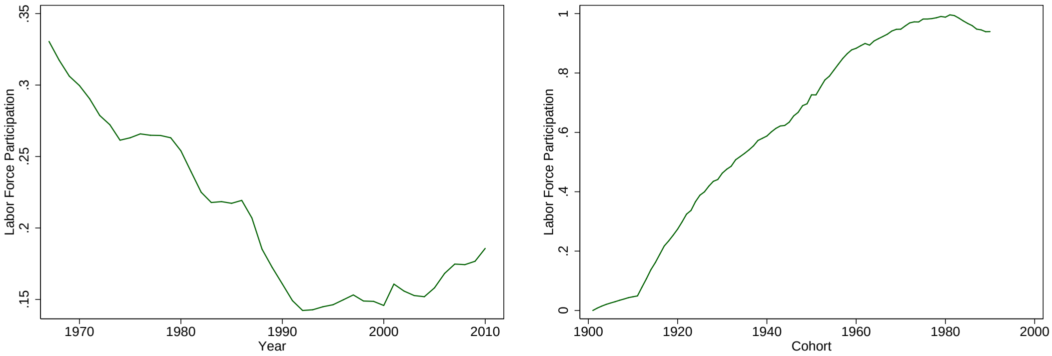
The top figure, panel (a), shows period and cohort effects when age effects are set to zero. Setting age effects to zero seems unrealistic in this context. The descriptive figures in Section 4 suggests increasing period effects. This approach show that the period effects decreased for most of the time period until the early 1990s, then started to increase during the bank crisis, and continued to increase during the financial crisis. Cohort effects are increased a lot, from almost zero to one. It is somewhat unrealistic that almost all of the increase in the labor force participation rate is due to cohort effects.

Panel (b) shows the effect of setting period effects to zero. This effectively removes the common exposure of social and historical shifts, resulting in a hump shaped age effect and an increasing cohort effect, which seems reasonable. On the other hand, removing period effects also removes the effect of women entering the labor market, which occurred over the last decades. The estimated cohort effect increases almost 30 percentage points, which suggests that some of the period effects may have been absorbed by the cohort effects.

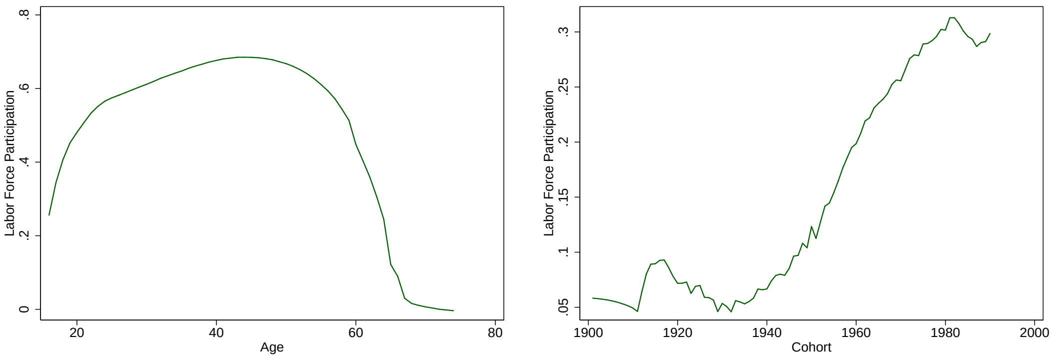
Panel (c) shows the effect of setting cohort effects equal to zero. Setting cohort effects to zero removes the effect of events that potentially affect different cohorts in different ways. The effect of entering the labor market during a recession, for instance, is removed. The result is a hump shaped age effect and an increasing period effect. The negative age effect at the end seems to have absorbed the effect of the elderly working less, which is a cohort effect.

Figure 8. Setting age, period, and cohort effects to zero

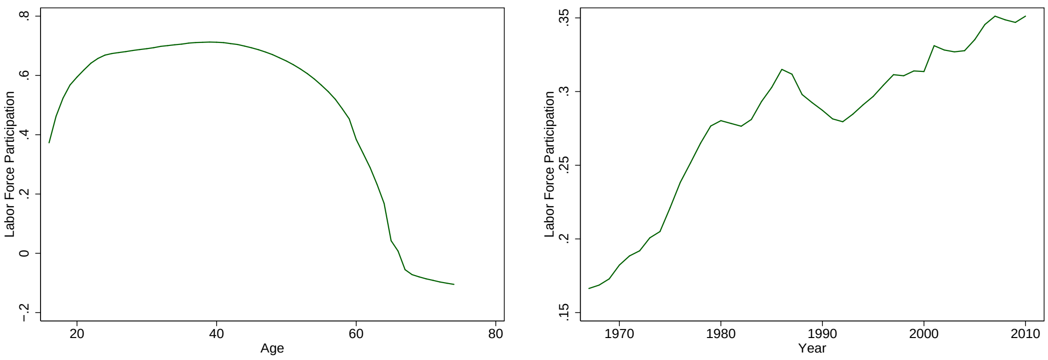
(a) Setting age effects to zero



(b) Setting period effects to zero



(c) Setting cohort effects to zero



Note: The estimated effect on labor force participation for men and women aged 15 to 74 years when setting age, period, or cohort effects to zero.
Estimated by using income data from the Norwegian tax administration.

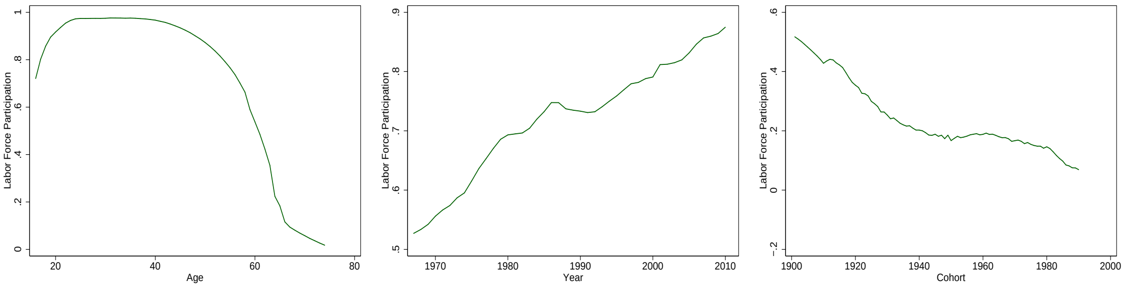
6.2 Constrained generalized linear models (CGLIMs)

The constrained generalized models approach can take the form of setting two adjacent ages to be equal. Figure 9 shows the result for three different examples. Panel a shows the result from setting age 29 equal to age 30. Age effects follow the expected hump shape, period effects increase by almost 40 percentage points, and cohort effects decrease by about 30 percentage points. The combined effect from age and cohort is then about 15 percentage points. In panel (b), age 39 is set equal to age 40. The results are almost identical to the results in panel (a). This suggests that the results are invariant to the choice of normalization. However, if age 49 is set equal to age 50, the results change significantly. These results are shown in panel (c). The period effects decrease by about 10 percentage points, and the cohort effects increase by almost 50 percentage points. This implies a combined effect of period and cohort of 40 percentage points.

This shows that the effects depend highly on which age groups that are set equal.

Figure 9. Setting adjacent ages equal

(a) Setting age 29=30



(b) Setting age 39=40

(c) Setting age 49=50

Notes: This figure shows the estimated age, period, and cohort effects on labor force participation for men and women aged 15 to 74 years when we set two adjacent age groups equal.
Estimated by using income data from the Norwegian Tax administration.

6.3 Deaton-Paxson normalization

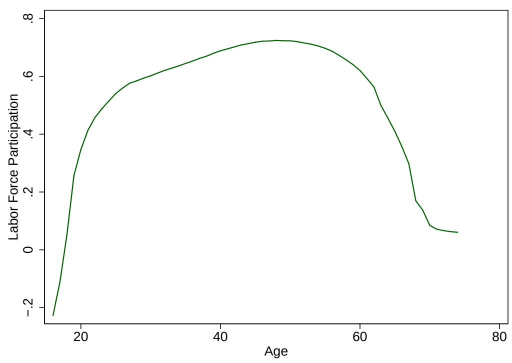
Figure 10 shows the result from Deaton-Paxson normalization.[19] The age effect follows the expected hump shape, consistent with the pattern given by the other solutions. Now the period effect is restricted so the fluctuations around the trend

[19] The Deaton-Paxson estimates are calculated using the ado-file -apc- for linear apc-models generated by O'Dea (2012)

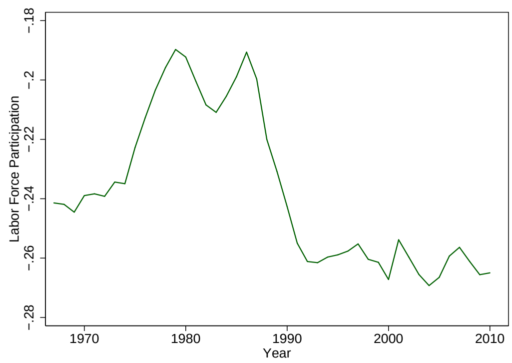
must sum to zero. As shown in panel (b), the fluctuations are above the trend in the first part and below in the second part. The variation is more systematic than a random variation around a trend (which is the restriction). This might be an indication that the restriction is problematic. The cohort effects are shown in panel (c). They increase by about 30 percentage points.

Figure 10. Deaton-Paxson normalization

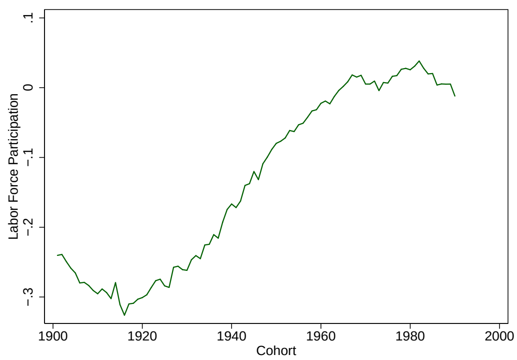
(a) Age effects



(b) Period effects



(c) Cohort effects



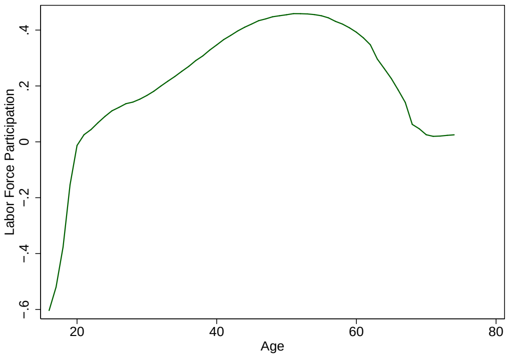
Notes: Age, period and cohort effects on labor force participation for men and women aged from 16 to 74 years, estimated by Deaton-Paxson normalization.
The Deaton-Paxson estimates are calculated using the ado-file -apc- for linear APC models generated by O'Dea (2012), using income data from the Norwegian tax administration.

Figure 11 shows the effects separately for men and women. The age profile for men follows the expected hump shape. The first two years, the age profile for women are negative, but then it also follows the expected hump shape. The period effect decreases for women, which does not fit very well with the female entrance to the labor force. For men the period effect fluctuates a lot, but the trend is flat.

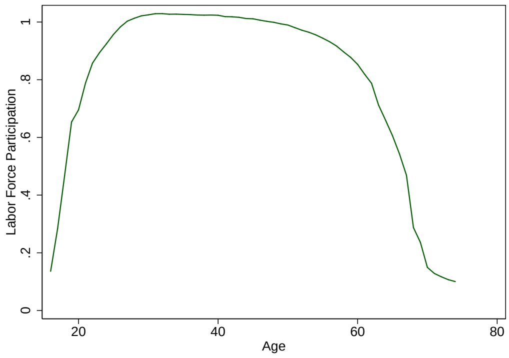
The female cohort effect increases by almost 70 percentage points. This is very high, and it seems that some of the period effect is absorbed by the cohort effect. This may be explained by the negative age effect for those younger than 20 years old and the negative period effect. For males the cohort effect decreases. One explanation for this decrease is the education effect; it has become more normal to take higher education. The latest cohorts are just above 20 years old in 2011 and are still in education. This means that they have a lower participation rate than older cohorts at the same age.

Figure 11. Deaton-Paxson estimates for men and women

(a) Age effects, women



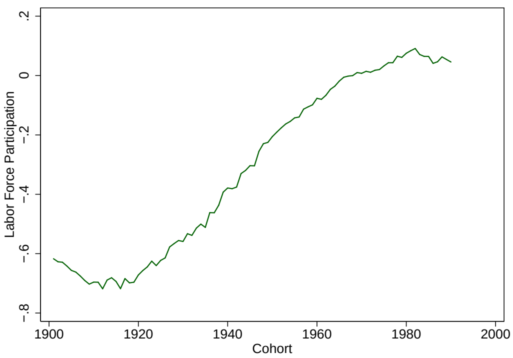
(b) Age effects, men



(c) Period effects, women

(d) Period effects, men

(e) Cohort effects, women

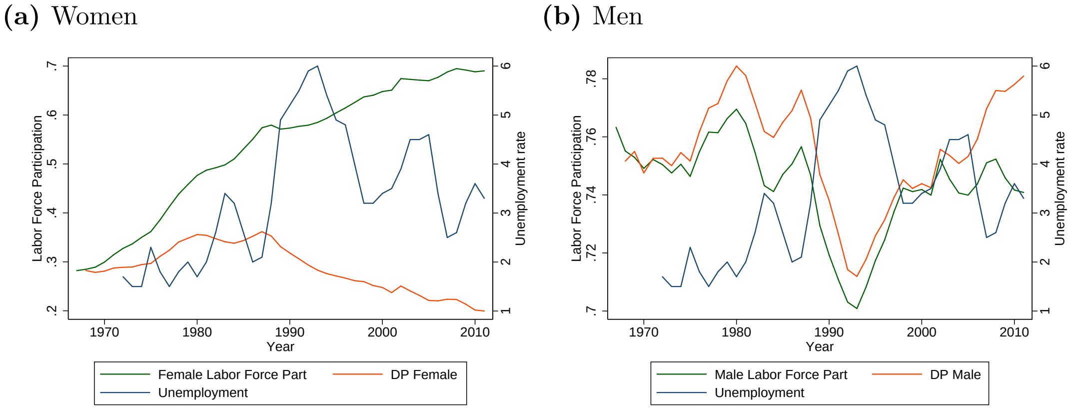


(f) Cohort effects, men

Notes: Age, period, and cohort effects on labor force participation for men and women aged 16 to 74 years, estimated by Deaton-Paxson normalization for men and women separately. The Deaton-Paxson estimates are calculated using the ado-file -apc- for linear APC models generated by O'Dea (2012), using income data from the Norwegian tax administration.

This method is not appropriate for estimating age, period and cohort effects for women, the reason is the clear break in the labor force participation rate for women. Figure 3 shows that the trend in female labor force participation is increasing more in the period from 1967 to 1990, then the trend is adjusted downward, causing problems using the Deaton-Paxson approach. As a result, the age effects are negative at younger ages. The period effect is negative, and some of the period effect seems to be pushed over as cohort effects, giving unrealistic estimates.

Figure 12. Deaton-Paxson period effects and the unemployment rate



Notes: This figure shows the labor force participation rate from the Labor Force Survey, the period effect estimated from Deaton-Paxson estimation and unemployment rates. a) female and b) male

Comparing the Deaton-Paxson estimates of the period effects and the labor force participation rate shows that the estimates for men follow the participation rate in the Labor Force survey quite well. It also inversely follows the unemployment rate, which is consistent with the assumption that people leave the labor force in economic downturns. The Deaton-Paxson period effects for women are develop in the opposite direction compared to the participation rate in the Labor Force Survey, and it do not correlate with the unemployment rate. This supports the argumentation above, stating that Deaton-Paxson estimation is not a good way to estimate age, period, and cohort effects for women.

6.4 Maximum entropy estimates

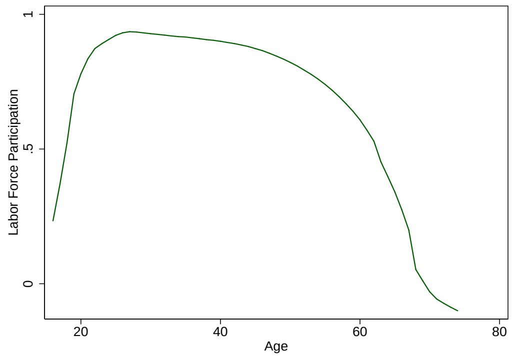
Figure 13 gives the result when using the maximum Entropy estimates.[20] Age and period effects seem to follow the expected pattern. The age effects follow the

[20] The maximum entropy estimates are calculated using the ado-file -apc- for linear apc-models generated by O'Dea (2012).

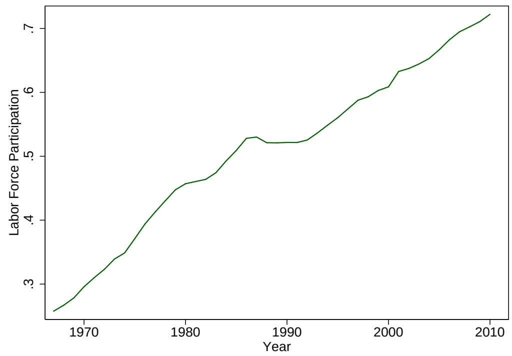
hump shape seen in the other approaches and period effects increase the whole time period. Cohort effects decrease, which is not the expected outcome considering the descriptive statistics, which shows increasing female cohort effects and constant male cohort effects.

Figure 13. Maximum entropy estimates

(a) Age effects



(b) Period effects



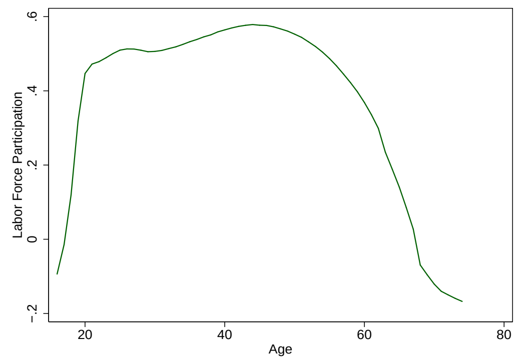
(c) Cohort effects

Notes: Age, period, and cohort effects on labor force participation for men and women aged 16 to 74 years estimated by the maximum entropy estimator. The maximum entropy estimates are calculated using the ado-file -apc- for linear APC models generated by O'Dea (2012), using income data from the Norwegian tax administration.

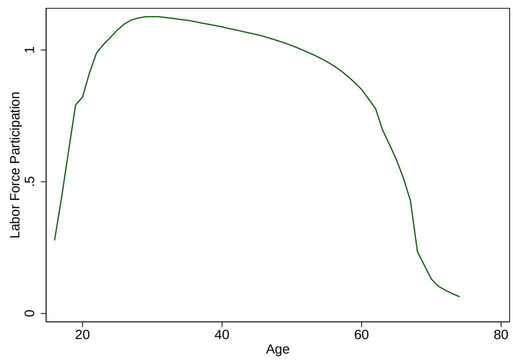
Separating the sample into men and women gives somewhat different results. The age effect is reasonable for both men and women and follows the expected hump shape. Female period effects increase the entire period, except for a flat period around the banking crisis in the beginning of the 1990s. The effect from the banking crisis is much more visible for men, with a large drop in labor force participation. Female cohort effects fluctuate a lot, but for the cohorts born between 1920 to 1950 the participation has increased a lot. This reflects the entrance of the housewives into the labor market. Then the cohort effect is increasing. The cohort effects for males decrease the whole period. These effects are shown in Figure 14.

Figure 14. Maximum entropy estimates for men and women

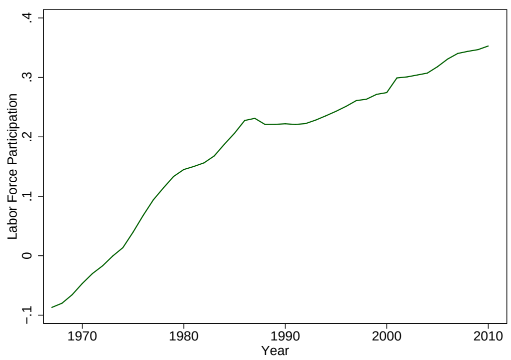
(a) Age effects, women



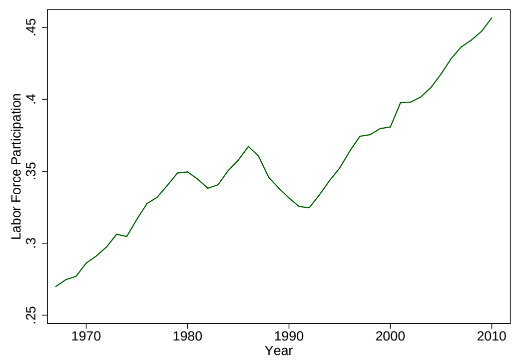
(b) Age effects, men



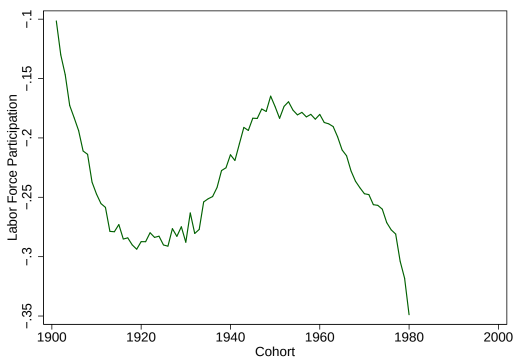
(c) Period effects, women



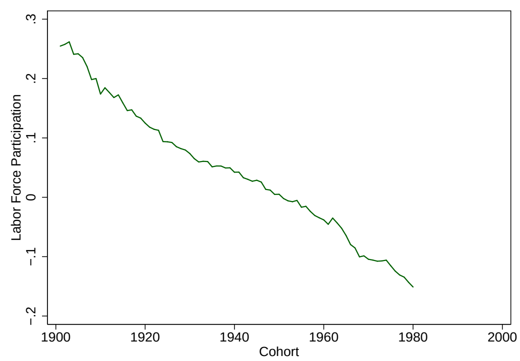
(d) Period effects, men



(e) Cohort effects, women



(f) Cohort effects, men

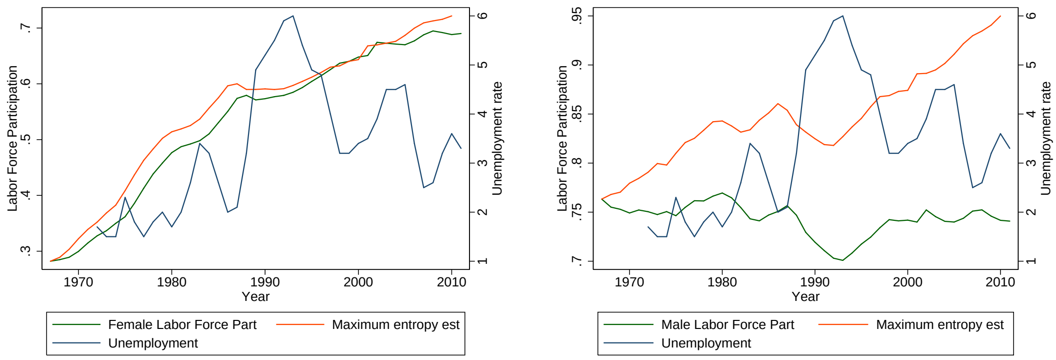


Notes: Age, period, and cohort effects on labor force participation for men and women aged 16 to 74 years estimated by the maximum entropy estimator for men and women separately. The maximum entropy estimates are calculated using the ado-file -apc- for linear APC models generated by O'Dea (2012), using income data from the Norwegian tax administration.

Figure 15 shows a comparison of the estimated period effects, labor force participation and unemployment from the Labor Force Survey separately for men and women. The plot shows that for men, the estimated period effect on labor force participation and unemployment rates are inversely related which is reasonable. However, the estimated period effect do not follow the actual labor force participation rate. The female plot shows a much better fit for, but the inverse relationship to unemployment are less distinct.

Figure 15. Maximum entropy estimates and labor force participation

(a) Women **(b)** Men



Notes: Labor force participation, maximum entropy estimates (period effects), and unemployment for (a) women and (b) men aged 16 to 64 years. The maximum entropy estimates are calculated using the ado-file -apc- for linear APC models generated by O'Dea (2012), using income data from the Norwegian tax administration. Unemployment and labor force participation from the Norwegian Labor Force Survey.

6.5 A proxy variable approach

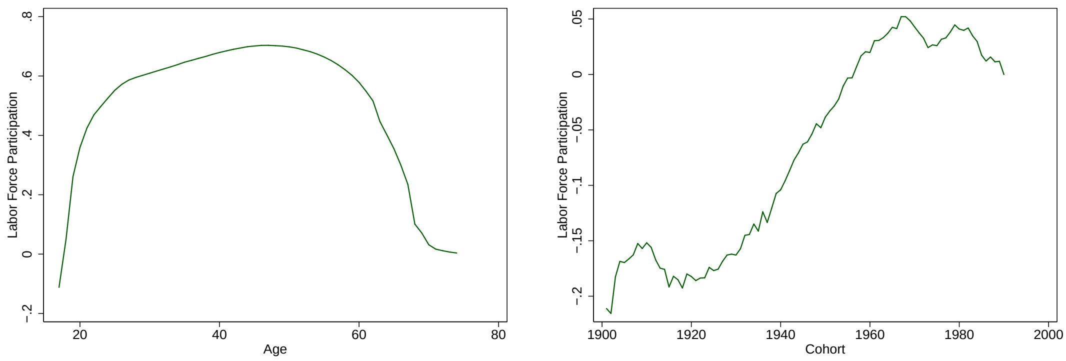
A final method to solve the APC problem is to use a proxy for time and for cohort.

A proxy for time effects

Unemployment rates can be used as a proxy for time. Time effects are supposed to reflect period-specific changes that affect all individuals in different cohorts. It is natural to believe that the unemployment rate will have an impact on labor force participation rates for all cohorts. Typically, labor force participation decreases when unemployment rates increase. When it gets harder to find a job, people tend to drop out of the labor force. The reason may be different among different age groups, but the effects is the same. Younger people tend to spend more time on education, those with kids stay home, and older people retire earlier. This effect is called the hidden unemployment (Borjas (2013), page 23).

Figure 16. Age and cohort effects when using unemployment for time

(a) Age effects **(b)** Cohort effects



Notes: Age and cohort effects on labor force participation for men and women aged 17 to 74 years, using unemployment as a proxy for time.
Estimated by using income data from the Norwegian tax administration.

Figure 16 shows the age effects and cohort effects when using unemployment as a proxy for time. Age effects follows the usual hump shape but cohort effects become very large. The proxy variable will capture only a part of the variation, and the redundant variation will be absorbed by the cohort effect, making the cohort effect seem very important.

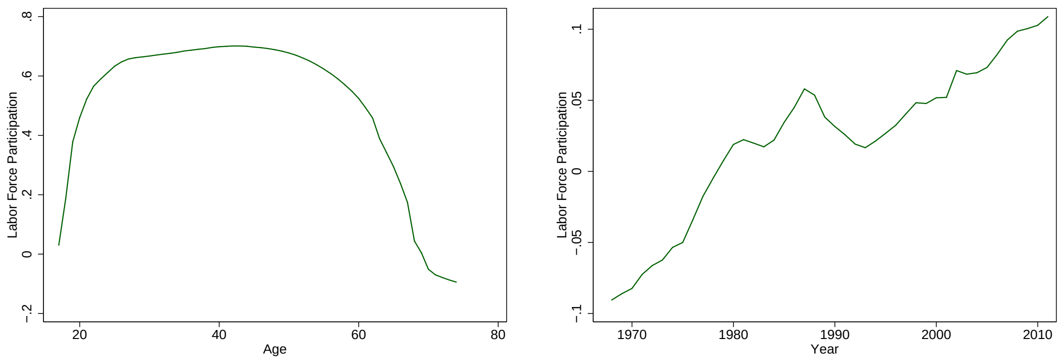
A proxy for cohort effects

GDP deviation from trend at age 25 can be used as a proxy for cohort effects. Cohort effects are supposed to reflect changes in society that affect different cohorts. Typically people finish their education around the age of 25. At this time they start to look for a job and enter the labor force. If the cohort enters the labor force during a recession or during an economic boom, this is believed to affect future earnings. Oreopoulos et al. (2012) estimated the long-term effects of entering the labor market in a recession and found that the cost of recession for new graduates is substantial and unequal. More advantaged graduates suffer less than less advantaged graduates because they shift to better firms more quickly.[21] This evidence suggests that one can rely on economic conditions during the early work years to model cohort effects. Accordingly, GDP deviations from trend can be used as a proxy for cohort effects.

[21] Firm quality is measured by firm size and average earnings among employees. The students are ranged due to predicted earnings based on college and major (measure of skill). More advantaged graduates has a higher predicted wage.

Figure 17. Age and period effects when using GDP deviation for cohort

(a) Age effects **(b)** Period effects



Notes: Age and cohort effects on labor force participation for men and women aged 17 to 74 years, using GDP deviation from trend at age 25 as a proxy for cohort. Estimated by using income data from the Norwegian tax administration.

Figure 17 shows the age and period effects when using GDP deviation from trend at age 25 as a proxy for cohort effects. Similar patterns as with a time proxy are found. When using GDP deviation at age 25 as a proxy for cohort, this captures only a part of the variation in cohort and some of the redundant variation is absorbed into the period effect, making the period effect look very important.

Comparing the proxy variables

Table 3 shows the estimated coefficients when a proxy-variable for either period or cohort is used. Unemployment at time t has a negative effect on labor force participation. In periods where there is an economic downturn, unemployment will rise and labor force participation will decline, which accords with previous evidence.

GDP deviation from trend at age 25 has a positive effect on labor force participation. The fact that the sign is positive indicates that more people enter the labor force when there are good economic conditions than leave the labor force in an economic downturn. Oreopoulos et al. (2012) found that higher unemployment at the time of graduation results in a lower wage. Given that lower wage reduces the incentives to stay in the labor force, this is consistent with my findings.

Table 3. Proxying period and cohort effects

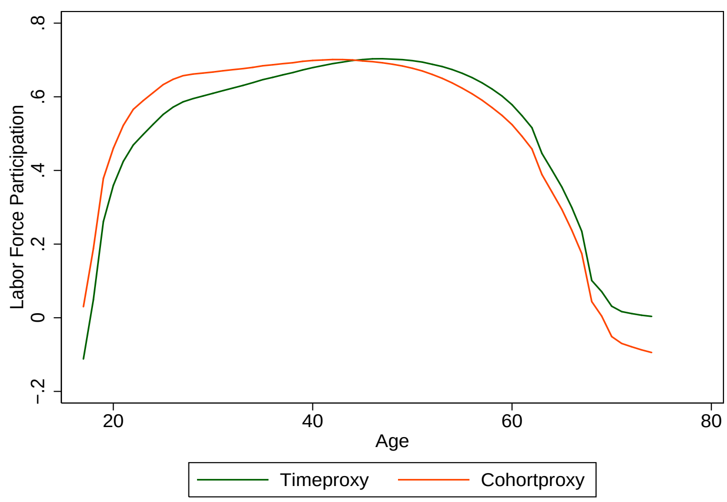
	Labor force participation	
Period proxy:		
Unemployment at t	-0.0047	-
	(0.0001)***	-
Cohort proxy:		
GDP deviation at age 25	-	0.2840
		(0.0015)***
Additional controls:		
Age indicators	✓	✓
Period indicators		✓
Cohort indicators	✓	
N	12 028 994	13 372 961

Notes: The table shows the estimated coefficients when we use a proxy for either period or cohort or both. The sample size differs because unemployment is only reported from 1972.

Estimated by using income data from the Norwegian tax administration.

Figure 18 compares the age effects using a proxy for cohort and time. The age effects estimated by the two different proxies follow a similar pattern. Proxying cohort gives a somewhat higher age effect at younger ages, while proxying period effects gives a somewhat higher age effect at older ages.

Figure 18. Comparing age patterns when using a proxy for either period and cohort effects



Notes: Age effects on labor force participation for men and women aged 17 to 74 years, using unemployment as a proxy for period and GDP deviation from trend as a proxy for cohort.
Estimated by using income data from the Norwegian tax administration.

7 Conclusion

This thesis, using income data from the Norwegian tax administration, examines the development in the labor force participation rate from 1967 to 2011. The focus is on disentangling age, period, and cohort effects. Since 1967 a major increase in labor force participation has taken place. The main reason for the increase is that women have entered the labor market. The participation rate is defined as those who have income at least equal to the basic amount of the Norwegian Social Insurance scheme. The data shows that the participation rate for women increases from 28 percent to 67 percent in the data set this thesis use; this is a somewhat larger increase than the participation rate in the Labor Force Survey owing to the definition of participation. The participation rate for men is almost constant, except for the increase since 2000 among older workers, aged 64 to 75 years.

The labor supply model described in section 3 can be used to explain the increase in female labor force participation. According to the model, higher wage will increase labor force participation if leisure is a normal good. During the 1970s it became more common to take higher education which also increased the wage for women in the labor force and increased the opportunity cost of staying home. This might be one reason for the increase in labor force participation among women.

Differences in the labor supply over the life cycle is a natural effect of age, but descriptive evidence suggest that also period of time and birth year has an important impact on the labor supply over time. The main challenge is to separate age, period, and cohort effects because of their linear relationship. To solve this problem, five different methods is presented. The first approach, setting age, period, or cohort to zero, suggests that age effects are the most important and that period and cohort effects seem to complement each other. The Constrained generalized linear model (CGLIM) approach, where adjacent ages are set equal, depends highly on which age groups that is chosen to be set equal and for that reason not is an appropriate method to solve the problem. The Deaton-Paxson approach suggests that period effects increase labor supply while cohort effects decrease labor supply for the total sample. A downward-sloping cohort effect seems unreasonable. Because of the clear break in the trend in labor supply of women, this is not an appropriate method to solve the problem. The maximum entropy method estimates positive period effects for both men and women. The cohort effect increases for females and decreases for males. The last approach is to use a proxy for either period or cohort effects. The unemployment rate is used as a proxy for period and finds that an increase in unemployment rate decreases the labor force participation rate. In this case, the cohort effect is very high, so it seems that some of the period effect is converted into a cohort effect. Also GDP deviation is used as a proxy for cohort effects. In this case, the period effect is very high, which confirms that the period and cohort effects are converted into the other when one is left out.

All methods show that the age effect is an important component of the changes in labor supply over the life cycle. They estimate approximately the same life cycle pattern, showing that the labor supply increase from the 20s and the 30s, when it is normal take education and get children, reach a top in the 50s and then the labor supply decline until reaching retirement age. The different methods also find distinct period and cohort effects but they do not agree in how large these effects are nor in which direction they are pointing. Considering the limitations of the different approaches the maximum entropy approach seems to be the most reliable approach, suggesting that female period effects increase the entire period, except for a flat period around the banking crisis in the beginning of the 1990s. The effect of the banking crisis is much more visible for men, with a large drop in labor force participation. Female cohort effects increase for cohorts born between 1920 and 1950, reflecting the entrance of the housewives into the labor market. The male cohort effects are decreasing the whole period. The plots of period effects and unemployment rates shows that for men especially, the estimated period effect on labor force participation and unemployment rates are inversely related. The female plot shows a similar story, but the effects are less distinct.

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